Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Friday 19 May 2023

Morning (Time: 2 hours)

Paper reference **4MA1/1H**

Mathematics A
PAPER 1H
Higher Tier

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over

International GCSE Mathematics

Formulae sheet – Higher Tier

Arithmetic series

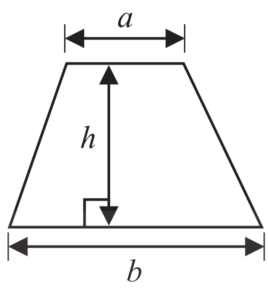
Sum to n terms, $S_n = \dfrac{n}{2}[2a + (n - 1)d]$

The quadratic equation

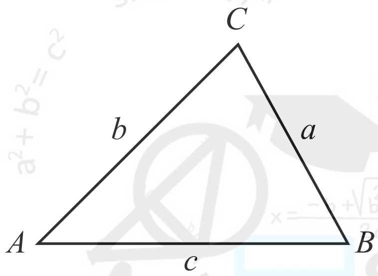
The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium $= \dfrac{1}{2}(a + b)h$



Trigonometry



In any triangle ABC

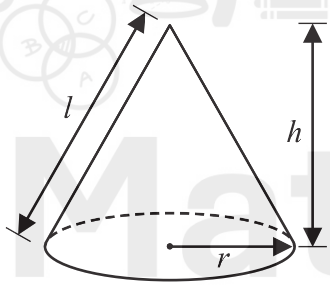
Sine Rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos A$

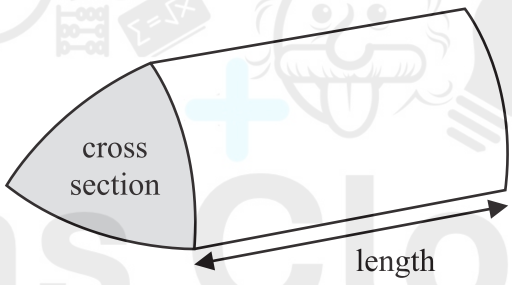
Area of triangle $= \dfrac{1}{2}ab\sin C$

Volume of cone $= \dfrac{1}{3}\pi r^2 h$

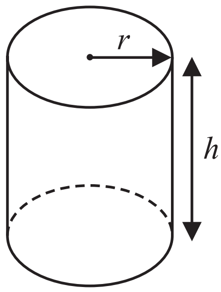
Curved surface area of cone $= \pi rl$



Volume of prism
= area of cross section × length

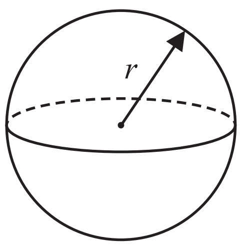


Volume of cylinder $= \pi r^2 h$
Curved surface area of cylinder $= 2\pi rh$



Volume of sphere $= \dfrac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Answer ALL TWENTY THREE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Last season, the number of goals scored by Arjun, by Simon and by Kath for their football team were in the ratios 2 : 5 : 8

Simon scored 12 more goals than Arjun.

Work out the number of goals scored by Kath.

Method 1

let x = the value of one share

Form an equation involving Simon and Arjun's ratios

Simon scored 12 more goals than Arjun, so the equation forms as:

$5x = 2x + 12$

Minus $2x$ from both sides of the equation.

$3x = 12$

divide both sides of the equation by 3

$x = 4$

Kath has 8 shares within the ratio

number of goals scored by Kath = value of 1 share × number of shares in the ratio

= 4 × 8

= 32 goals scored by Kath

32

(Total for Question 1 is 3 marks)

Method 2

Using Arjun and Simon's ratios

Arjun : Simon

2 : 5

We must multiply the ratio values until Simon's value is 12 more than Arjun's, as we are told that Simon scores 12 more goals than Arjun.

2 : 5 (×2)

4 : 10 ← 10 − 4 = 6, so not the desired difference (12).

Therefore, multiply by 2 again.

4 : 10 (×2)

8 : 20 ← 20 − 8 = 12, this is the desired difference

This method is the same as multiplying by 4.

To find total scored by Kath:

4 × 8 = 32 goals scored by Kath.

2 The table gives information about the number of minutes that Abby spent walking each day in September.

	Number of minutes (M)	Frequency	Midpoint	midpoint x frequency
1	$0 < M \leqslant 30$	5	15	75
2	$30 < M \leqslant 60$	6	45	270
3	$60 < M \leqslant 90$	8	75	600
4	$90 < M \leqslant 120$	9	105	945
5	$120 < M \leqslant 150$	2	135	270

Work out an estimate for the total number of minutes that Abby spent walking in September.

Step 1

To calculate the midpoint:

1) $\frac{0+30}{2} = 15$

2) $\frac{30+60}{2} = 45$

3) $\frac{60+90}{2} = 75$

4) $\frac{90+120}{2} = 105$

5) $\frac{120+150}{2} = 135$

Step 2

To calculate midpoint x frequency

1) $5 \times 15 = 75$

2) $6 \times 45 = 270$

3) $8 \times 75 = 600$

4) $9 \times 105 = 945$

5) $2 \times 135 = 270$

Step 3

Calculate the total number of minutes Abby spent walking.

$75 + 270 + 600 + 945 + 270 = 2160$ minutes

2160 minutes

(Total for Question 2 is 3 marks)

3 Nanette buys 60 notebooks for a total cost of 780 dirhams.

Nanette sells 70% of the notebooks for 22 dirhams each.
She sells the remaining notebooks for 19 dirhams each.

Work out Nanette's percentage profit.
Give your answer correct to 3 significant figures.

Method 1

Calculate the income made from 70% of notebooks sold at 22 dirhams.

$0.7 \times 60 \times 22 = 924$ dirhams

To calculate the income made from remaining notebooks sold at 19 dirhams

$(1 - 0.7) \times 60 \times 19$

$0.3 \times 60 \times 19 = 342$ dirhams

Calculate the percentage profit:

$\frac{924 + 342 - 780}{780} \times 100$ or $\frac{924 + 342}{780} \times 100 - 100$

$= 62.30769231$

Rounding to 3 significant figures.

$= 62.3\%$

Method 2

Calculate the profit made for 70% of notebooks sold at 22 dirhams.

$0.7 \times 60 \times (22 - \frac{780}{60}) = 378$

To calculate the profit made for the remaining notebooks sold at 19 dirhams.

$(1 - 0.7) \times 60 \times (19 - \frac{780}{60}) = 108$

To calculate the percentage profit:

$\frac{378 + 108}{780} \times 100$

$= \frac{486}{780} \times 100$

$= 62.30769231$

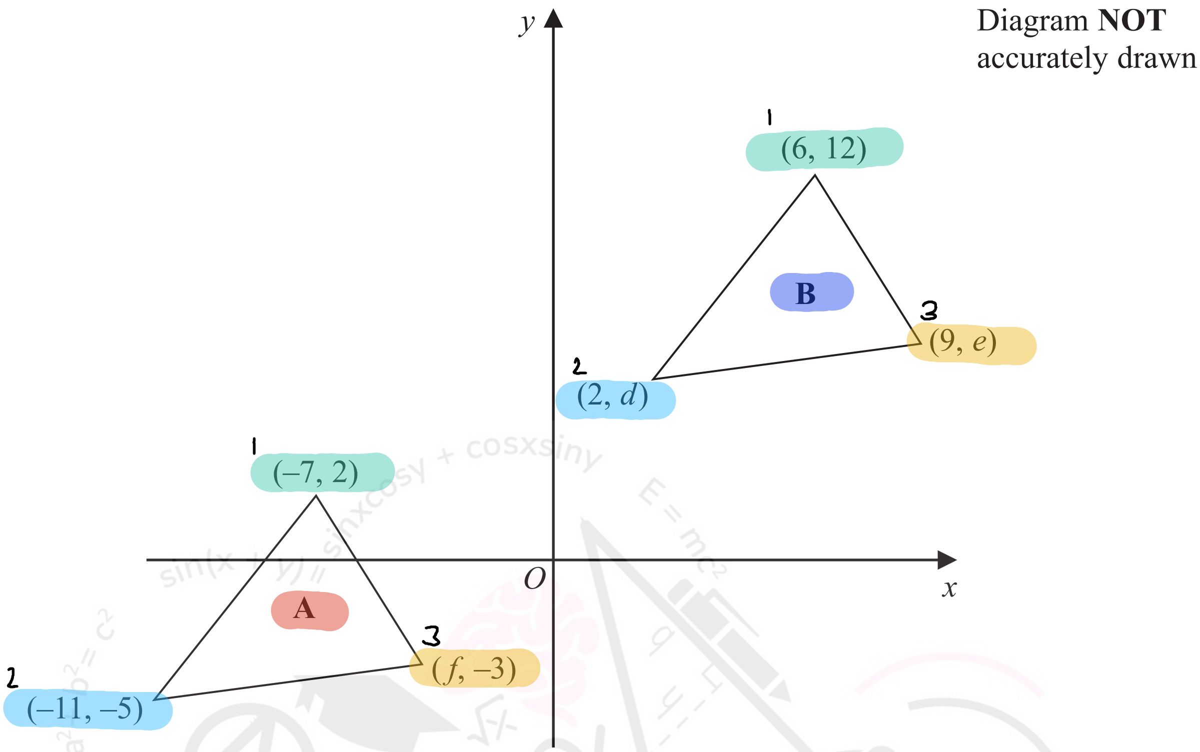
Rounding to 3 significant figures:

$= 62.3\%$

62.3 %

(Total for Question 3 is 4 marks)

4 The diagram shows a sketch of triangle **A** and triangle **B** on a coordinate grid.



(a) Given that triangle **A** has been translated to give triangle **B**, work out the value of d, the value of e and the value of f

1) $\begin{pmatrix}-7\\2\end{pmatrix} \rightarrow \begin{pmatrix}6\\12\end{pmatrix}$

from -7 to $6 = 6-(-7) = 13$
from 2 to $12 = 12-2 = 10$
Therefore translation = 13 right, 10 up

Note: for the translation, the top figure is a translation right / left, the bottom figure is a translation up / down

e.g: $\begin{pmatrix}1\\2\end{pmatrix}$ = 1 right, 2 up
$\begin{pmatrix}-1\\-2\end{pmatrix}$ = 1 left, 2 down

2) $\begin{pmatrix}-11\\-5\end{pmatrix} \rightarrow \begin{pmatrix}2\\d\end{pmatrix}$

To get from -5 to d: $-5+10 = 5$
$d = 5$

3) $\begin{pmatrix}f\\-3\end{pmatrix} \rightarrow \begin{pmatrix}9\\e\end{pmatrix}$

To get from 9 to f: $9-13 = -4$ (-13 instead of $+13$ as going from B to A instead of A to B)
$f = -4$
To get from -3 to e: $-3+10 = 7$
$e = 7$

$d =$ 5

$e =$ 7

$f =$ −4

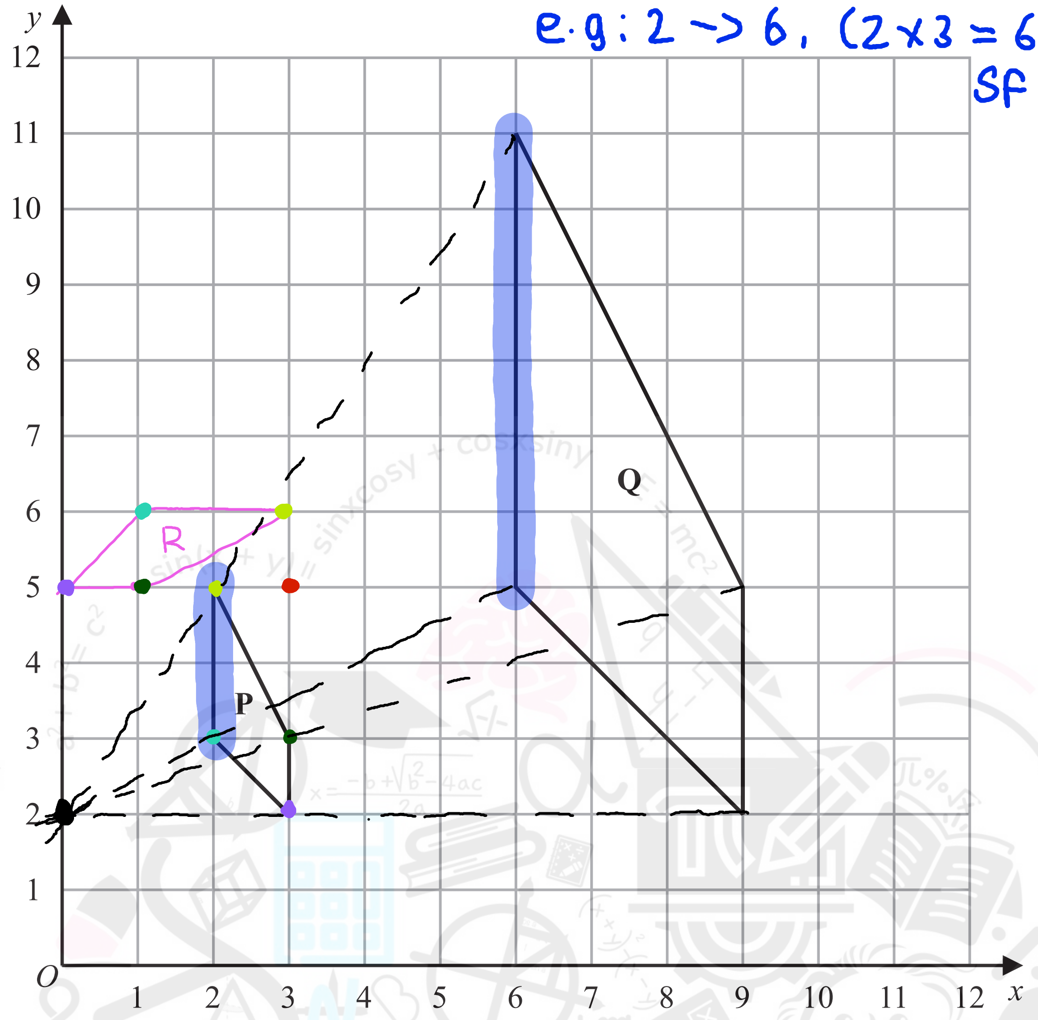
(3)

P 7 2 7 9 0 A 0 6 2 8

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

The diagram shows shape **P** and shape **Q** drawn on a grid.

Note: To find the scale factor, assess the difference between 2 sides.
e.g: 2 -> 6, (2×3 = 6), so SF = 3



(b) Describe fully the single transformation that maps shape **P** onto shape **Q**

Enlargement, scale factor 3, centre (0,2)

Note: As from P to Q, the shape increases in size.

(3)

(c) On the grid above, rotate shape **P** 90° clockwise about (3, 5)
Label your shape **R**

(2)

(Total for Question 4 is 8 marks)

P 7 2 7 9 0 A 0 7 2 8

Turn over

5 The diagram shows a shaded shape *AEBCD* made by removing triangle *AEB* from rectangle *ABCD*

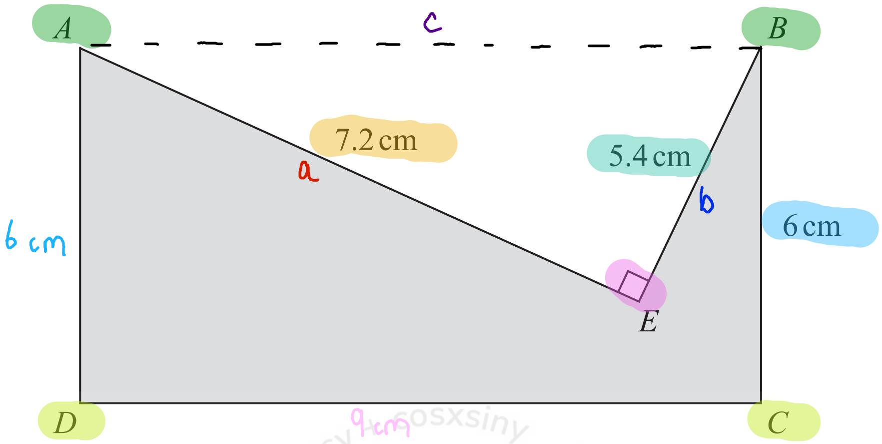


Diagram **NOT** accurately drawn

$AE = 7.2$ cm $\quad BE = 5.4$ cm $\quad BC = 6$ cm $\quad$ angle $AEB = 90°$

Work out the perimeter of the shaded shape.

BC = AD, so AD = 6cm

To find AB, use pythagoras on triangle AEB.

$a^2 + b^2 = c^2$

$7.2^2 + 5.4^2 = c^2$

$51.84 + 29.16^2 = c^2$

$c^2 = 81$

$c = \sqrt{81}$

$c = 9$ cm

$c = AB$

$AB = 9$ cm

$AB = DC$

so $DC = 9$ cm

$a + b + 6 + 6 + 9 =$ perimeter

$7.2 + 5.4 + 6 + 6 + 9 = 33.6$ cm

33.6 cm

(Total for Question 5 is 4 marks)

6 (a) Simplify $(2c^4d^7)^3$

cubed outside the bracket, so take $2^3 = 8$.
Indices rules tells us that the 3 outside the bracket means to multiply both power terms by 3.
$4 \times 3 = 12$
$7 \times 3 = 21$
So simplification $= 8c^{12}d^{21}$

$8c^{12}d^{21}$ ....

(2)

(b) Find the value of $5y^0$ where $y > 0$

Note: Any term to the power of $0 = 1$

$\therefore$

5 ....

(1)

(c) Factorise fully $16a^2b^3 + 20a^3b$

The highest common factor of 16 and 20 is 4.
The highest common factor of a^2b^3 and a^3b is a^2b
Therefore, $4a^2b$ can come out of the bracket.

$= 4a^2b(4b^2 + 5a)$

$4a^2b(4b^2 + 5a)$ ....

(2)

(d) (i) Factorise $x^2 + 9x - 22$

Need to find 2 figures which add to get 9, and multiply to get -22.
Using 2 and -11
$-2 + 11 = 9$
$-2 \times 11 = -22$
Factorised: $(x+11)(x-2)$

$(x+11)(x-2)$ ....

(2)

(ii) Hence solve $x^2 + 9x - 22 = 0$

$(x+11)(x-2) = 0$

$x + 11 = 0$
$x = -11$

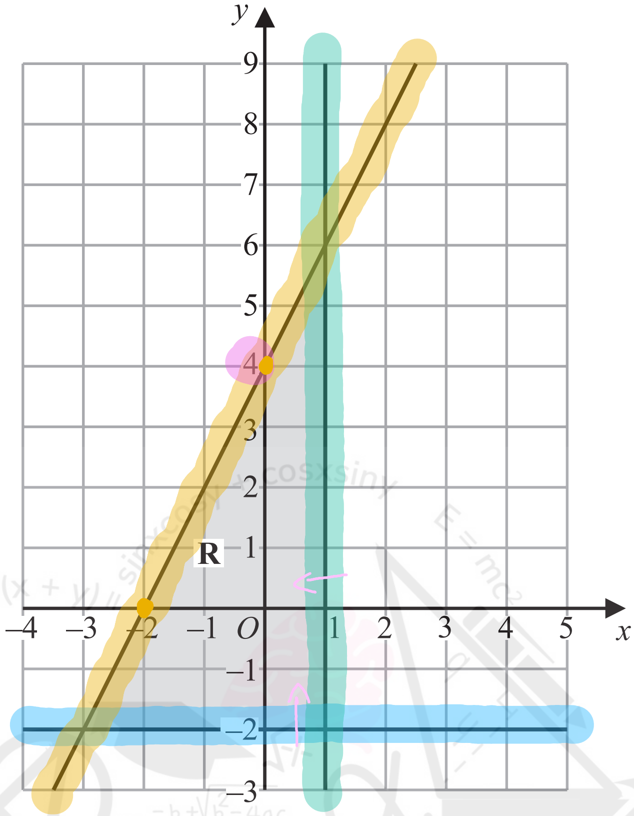
$x - 2 = 0$
$x = 2$

$-11, 2$ ....

(1)

(Total for Question 6 is 8 marks)

7



The region **R**, shown shaded in the diagram, is bounded by three straight lines.

Find the inequalities that define **R**

The line is horizontal, so it is the x axis.
It is a solid line on -2, so $y \geq -2$. Note: the shape is shaded towards figures greater than -2. $y \geq -2$

The line is vertical, so it is the y axis.
It is a solid line on 1, so $x \leq 1$. Note: the shape is shaded towards figures less than 1, so $x \leq 1$

To find the gradient, take 2 points from the graph.
$(-2, 0)$ and $(0, 4)$.
$x_1 \; y_1 \qquad x_2 \; y_2$

Plug into gradient formula: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{0-(-2)} = \frac{4}{2} = 2$
$m = 2$

Plug into $y = mx + c$
$y = 2x + c$

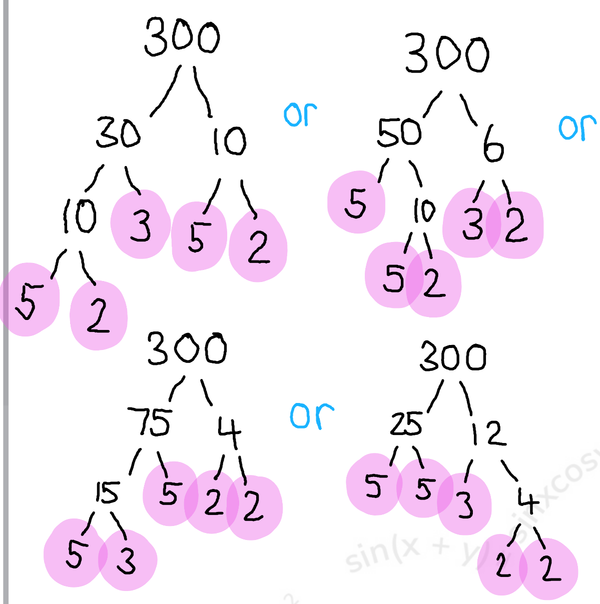
c = the point the line crosses the y axis.
so $c = 4$
so $y \leq 2x + 4$

$x \leq 1$
$y \geq -2$
$y \leq 2x + 4$

(Total for Question 7 is 4 marks)

Note: The shape is shaded towards figures less than -2.

8 (a) Write 300 as a product of its prime factors.
Show your working clearly.



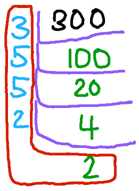
Or use the ladder method:

write 300 with a L: $\lfloor 300$

Find a factor and write it on the outside of the ladder $3 \lfloor 300$

Divide the 300 by the factor and write the result under each. $3 \lfloor 300$ 100

Draw another L and repeat the process until there is no number that 300 can be divided by.



The remaining outside numbers are the prime factors.

Prime factors of 300 $= 2 \times 2 \times 3 \times 5 \times 5$

$2 \times 2 \times 3 \times 5 \times 5$

(2)

$A = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$B = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

(b) Find the lowest common multiple (LCM) of $5A$ and $7B$
Show your working clearly.

$5A = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

$5A = 2^3 \times 3^2 \times 5^2$

$7B = 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7$

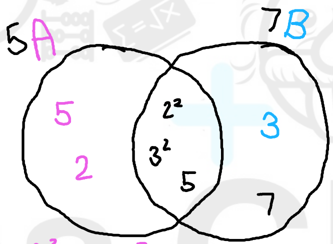
$7B = 2^2 \times 3^3 \times 5 \times 7$

$LCM = 2^3 \times 3^3 \times 5^2 \times 7$

$LCM = 37,800$

Note: we want to find the lowest power of each value to calculate LCM, which is in contrast to calculating HCF (highest common factor).

or use a venn diagram:



$5A = 2^3 \times 3^2 \times 5^2$

$7B = 2^2 \times 3^3 \times 5 \times 7$

common terms to go in the middle of the venn diagram $= 2^2, 3^2, 5$

The reminder go in the respective A and B sections

Multiplying what is in the venn
$= 2^2 \times 3^2 \times 5 \times 3 \times 7 \times 2 \times 5 = 37,800$

37 800

(2)

(Total for Question 8 is 4 marks)

9 Solve the simultaneous equations

Note: the same sign is present, so the equations must be subtracted.

1 $2x + 9y = 14.5$
2 $7x + 3y = 8$

Show clear algebraic working.

1) $7x + 3y = 8 \quad (\times 3)$

we multiply by 3 to get the same value (9) infront of the y's so we can solve simultaneously.

$= 21x + 9y = 24$
2) $2x + 9y = 14.5 \quad -$

$= 19x \qquad = 9.5$

we must have the same value infront of the x's or y's in order to solve simultaneously.

$19x = 9.5$
$x = \frac{9.5}{19}$
$x = 0.5$

Plug the x value back into equation 1 to find y.

$2x + 9y = 14.5$
$2(0.5) + 9y = 14.5$
$1 + 9y = 14.5$
$9y = 13.5$
$y = 1.5$
$x = 0.5$

$x =$ 0.5

$y =$ 1.5

(Total for Question 9 is 3 marks)

10 Here are the test marks of 15 students.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
7	10	14	15	16	17	18	19	20	20	23	25	30	39	40

Find the interquartile range of these marks.

Interquartile range = upper quartile (75%) - lower quartile (25%)

$15 \times 0.75 = 11.25^{th}$ value $\approx 12^{th}$ value of the data set

$15 \times 0.25 = 3.75^{th}$ value $\approx 4^{th}$ value of the data set.

12^{th} value $= 25$
4^{th} value $= 15$
$25 - 15 = 10$

Interquartile range = 10

10

(Total for Question 10 is 2 marks)

11 The curve **C** has equation $y = 4x^3 + x^2 - 20x$

(a) Find $\frac{dy}{dx}$

Note: Differentiation = Multiply by the power, −1 from the power.

$4x^3 = 12x^2$

$x^2 = 2x$

$-20x = -20$

$\frac{dy}{dx} = 12x^2 + 2x - 20$

$\frac{dy}{dx} = 12x^2 + 2x - 20$

(2)

(b) Find the x coordinates of the points on **C** where the gradient is 4
Show clear algebraic working.

$12x^2 + 2x - 20 = 4$

$12x^2 + 2x - 24 = 0$

$(6x - 8)(2x + 3) = 0$

$(x - \frac{8}{6})(x + \frac{3}{2}) = 0$

$x = \frac{8}{6}, \; x = -\frac{3}{2}$

$x = \frac{4}{3}, \; x = -\frac{3}{2}$

or

$(3x - 4)(2x + 3) = 0$

$(x - \frac{4}{3})(x + \frac{3}{2}) = 0$

$x = \frac{4}{3}, \; x = -\frac{3}{2}$

$a = 12 \quad b = 2 \quad c = -24$

or $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-2 \pm \sqrt{(2)^2 - 4(12)(-24)}}{2(12)}$

$x = \frac{-2 + \sqrt{(2)^2 - 4(12)(-24)}}{2(12)} \rightarrow x = \frac{4}{3}$

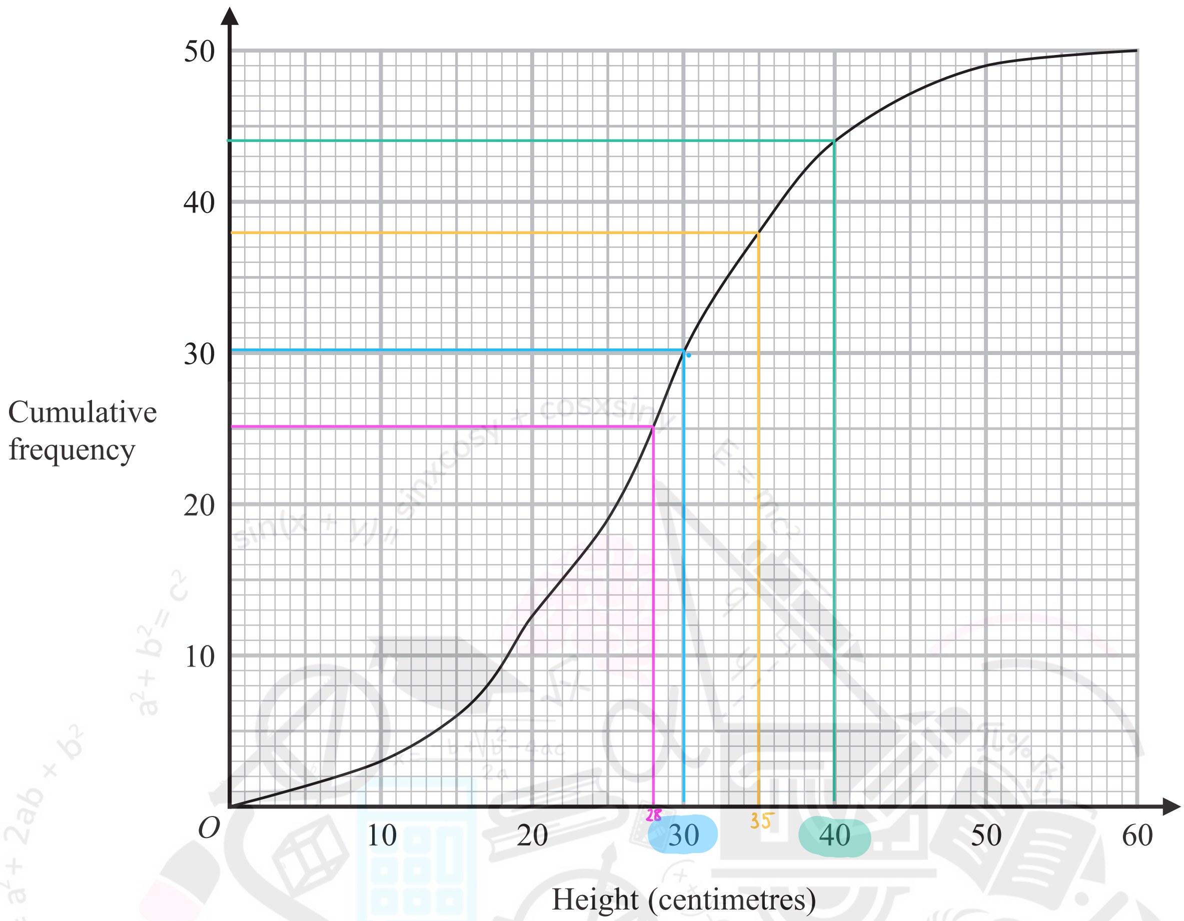
$x = \frac{-2 - \sqrt{(2)^2 - 4(12)(-24)}}{2(12)} \rightarrow x = -\frac{3}{2}$

$x = \frac{4}{3}, -\frac{3}{2}$

(4)

(Total for Question 11 is 6 marks)

12 The cumulative frequency graph shows information about the heights, in centimetres, of 50 plants in a flowerbed.



(a) Use the graph to find an estimate for the median height of these plants.

28 centimetres

(1)

(b) Use the graph to find the frequency for the class interval $30 < \text{Height} \leqslant 40$

$30 - 44 = 14$

14

(1)

(c) Use the graph to find an estimate for the number of plants with a height greater than 35 centimetres.

$50 - 38 = 12$

12

(2)

(Total for Question 12 is 4 marks)

13 A is the point with coordinates (-5, 12)

x_1 y_1

B is the point with coordinates (19, -48)

x_2 y_2

Find an equation of the straight line that passes through the points A and B

Equation of a line $= y = mx + c$

m = the gradient

Equation to find the gradient : $\frac{y_1 - y_2}{x_1 - x_2} = \frac{12 - (-48)}{-5 - 19} = 2.5$

$m = -2.5$

To find c, plug in the known values into $y = mx + c$

$12 = (-2.5)(-5) + c$

$12 = 12.5 + c$

$c = -0.5$

Equation : $y = -2.5x - 0.5$

$y = -2.5x - 0.5$

(Total for Question 13 is 3 marks)

14 Factorise fully $50g^2 - 18$

$2(25g^2 - 9)$ or $(10g + 6)(5g - 3)$ or $(5g + 3)(10g - 6)$ or $(5g + 3)(5g - 3)$

For all possibilities above : $2(5g \pm 3)(5g \pm 3)$ or $2(5g - 3)^2$

$= 2(5g + 3)(5g - 3)$

$2(5g + 3)(5g - 3)$

(Total for Question 14 is 3 marks)

15 (a) $\sqrt{2} \div \dfrac{8^3}{16^{\frac{3}{2}}} = 2^n$

Work out the value of n
Show your working clearly.

To solve, make all figures into the form : 2^x

$\sqrt{2} = 2^{\frac{1}{2}}$

$8^3 = 2^{3\times3} = 2^9$

$16^{\frac{3}{2}} = 2^{3\times2} = 2^6$

$\rightarrow 2^{\frac{1}{2}} \div \dfrac{2^9}{2^6} = 2^n = 2^{\frac{1}{2}} \div 2^3 = 2^{2.5}$, therefore $n = 2.5$

Note: indicies rules tells us that when dividing terms, the powers are subtracted. → subtracting the powers gives : $9 - 6 = 3$

$n =$ 2.5

(3)

(b) Find 4% of 4.5×10^{157}
Give your answer in standard form.

$4\% = 0.04$

$0.04 = 4 \times 10^{-2} \times 4.5 \times 10^{157}$

$4 \times 4.5 = 18$

$10^{-2} \times 10^{157} = 10^{155}$

$= 18 \times 10^{155}$

$= 1.8 \times 10^{156}$

Note : To be in standard form, the figure must be between 1 and 10.

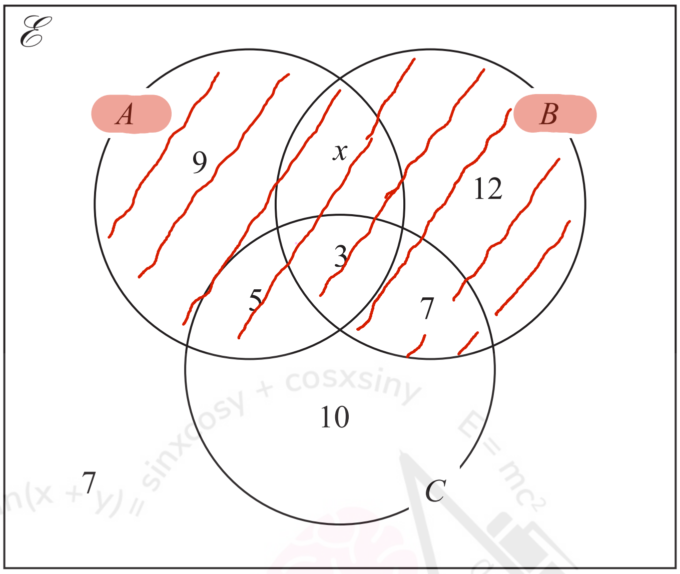
1.8×10^{156}

(3)

(Total for Question 15 is 6 marks)

16 The Venn diagram shows a universal set $\mathscr{E}$ and sets A, B and C

The numbers and the letter x represent **numbers** of elements.



Given that $n(A \cup B) = 42$

(a) find the value of x

$P(A) + P(B) = P(A \cup B)$

$9 + 5 + 3 + 7 + 12 + x = 42$

$36 + x = 42$

$x = 42 - 36$

$x = 6$

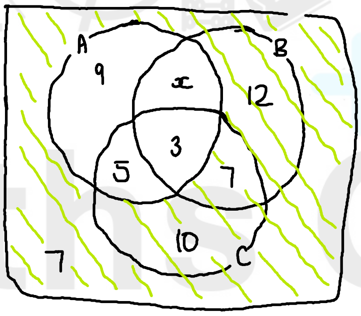
$x =$ 6

(1)

(b) Find $n(A')$

(A') = everything not in A.

$7 + 7 + 10 + 12 = 36$



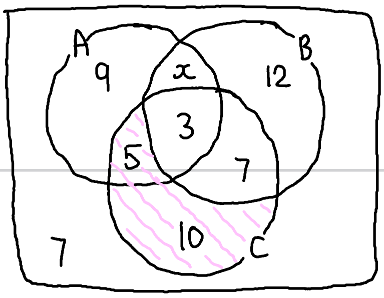
36

(1)

(c) Find $n(B' \cap C)$

$(B' \cap C)$ = Everything in C, that is not also in B.

$= 5 + 10 = 15$



15

(1)

(Total for Question 16 is 3 marks)

17 The functions g and h are such that

$$g(x) = \frac{11}{2x-5}$$

$$h(x) = x^2 + 4 \qquad x \geqslant 0$$

(a) What value of x must be excluded from any domain of g?

$\frac{11}{2x-5}$ when $x = 2.5$: $\frac{11}{2(2.5)-5} = \frac{11}{0}$ ← impossible to divide by 0

2.5

(1)

(b) Solve gh(x) = 1

$gh(x) = 1 \quad = \frac{11}{2x-5} \times h = 1$

Note: plug in the equation for h wherever an x is present in $\frac{11}{2x-5}$

$\frac{11}{2(x^2+4)-5} = 1$

$\frac{11}{2x^2+8-5} = 1$

$\frac{11}{2x^2+3} = 1$

$11 = 1(2x^2+3)$

$11 = 2x^2+3$

$2x^2 = 8$

$x^2 = 4$

$x = \sqrt{4}$

$x = 2$

2

(3)

(Total for Question 17 is 4 marks)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 7 2 7 9 0 A 0 1 8 2 8

18 The incomplete table and incomplete histogram give information about the times, in minutes, that 140 people waited at a station for a train.

Time (*t* minutes)	**Frequency**
$0 < t \leqslant 5$	23
$5 < t \leqslant 15$	25
$15 < t \leqslant 30$	60
$30 < t \leqslant 40$	18
$40 < t \leqslant 60$	14

Note: 10 small squares = 1 person

1) Calculate what 1cm square represents

$140 - (23 + 18 + 14)$

$140 - 55 = 85$

$85 \div 34 = 2.5$

number of 1cm squares on the grid.

1 cm square represents 2.5 people

2) Complete the table

2.5×24 squares $= 60$

2.5×10 squares $= 25$

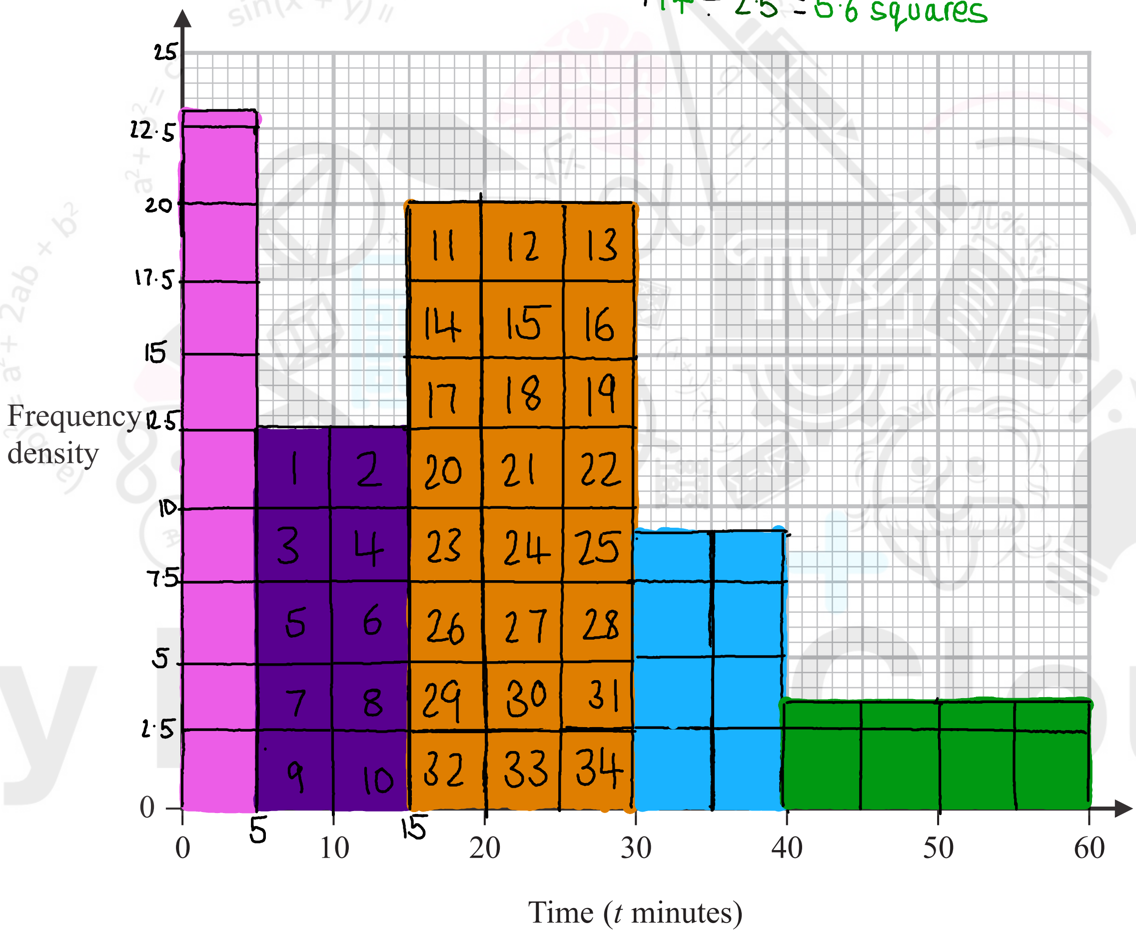
3) Complete the histogram

To omplete the histogram:

$23 \div 2.5 = 9.2$ squares

$18 \div 2.5 = 7.2$ squares

$14 \div 2.5 = 5.6$ squares



Complete the table and the histogram.

(Total for Question 18 is 4 marks)

P 7 2 7 9 0 A 0 1 9 2 8

19

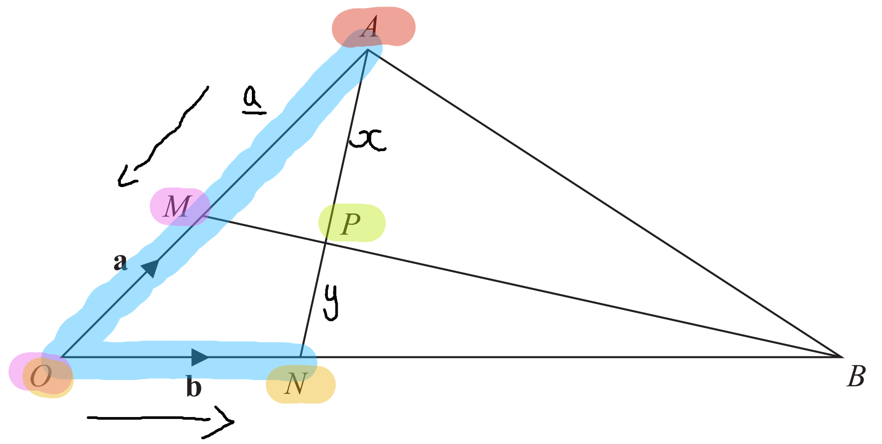


Diagram **NOT** accurately drawn

OMA, *ONB*, *MPB* and *NPA* are straight lines.

M is the midpoint of *OA*

$ON : NB = 1 : 5$

$\overrightarrow{OM} = \mathbf{a}$ $\quad \overrightarrow{ON} = \mathbf{b}$

(a) Find in terms of **a** and **b** the vector $\overrightarrow{AN}$

$\overrightarrow{OM} = \underline{a}$: To get from A to O, the distance is 2 lots of $\underline{a}$, against the vector direction, $= -2\underline{a}$.

$\overrightarrow{ON} = \underline{b}$: To get from O to N, the distance is 1 lot of $\underline{b}$, with the vector direction $= +1\underline{b}$, which can be written as: $+\underline{b}$.

$-2\underline{a} + \underline{b}$

(1)

(b) Use a vector method to find the ratio $AP : PN$

$\overrightarrow{AP} = x(\overrightarrow{AN})$

$\overrightarrow{AP} = x(-2\underline{a} + \underline{b})$ ①

$\overrightarrow{AP} = \overrightarrow{AM} + \overrightarrow{MP}$

$\overrightarrow{AP} = -\underline{a} + y(-\underline{a} + 6\underline{b})$

$x(-2\underline{a} + \underline{b}) = -\underline{a} + y(-\underline{a} + 6\underline{b})$

$-2\underline{a}x + \underline{b}x = -\underline{a} - \underline{a}y + 6\underline{b}y$

$\underline{b}x = 6\underline{b}y$

$x = 6y$ $\quad) \div \underline{b}$

$-2\underline{a}x = -\underline{a} - \underline{a}y$ $\quad) \div \underline{a}$

$-2x = 1 - y$

P 7 2 7 9 0 A 0 2 0 2 8

Plug in "$x = 6y$" into above equation.

$-2(6y) = 1 - y$

$-12y = 1 - y$

$-11y = 1$

$y = \frac{1}{11}$

$x = 6y$

$x = \frac{6}{11}$

If $\overrightarrow{AN} = 1$, this can be expressed as $\frac{11}{11}$.

so $y = \frac{11}{11} - \frac{6}{11}$

$y = \frac{5}{11}$

$x = \overrightarrow{AP}$, $y = \overrightarrow{PN}$

$\overrightarrow{AP} : \overrightarrow{PN}$

$= \frac{6}{11} : \frac{5}{11}$

$= 6 : 5$

$AP : PN =$ 6 : 5

(4)

(Total for Question 19 is 5 marks)

Turn over for Question 20

20 The sum of the first 80 terms of an arithmetic series, S, is 470

The 75th term of S is 14.5

The sum of the first X terms of S is 171

Work out the value of X
Show your working clearly.

Formula for the sum: $\frac{n}{2}[2a + (80-1)d]$ = the sum of the series

$\frac{80}{2}[2a + (80-1)d] = 470$

$\frac{80}{2}[2a + 79d] = 470$

$80[2a + 79d] = 940$

$\div 10$ $160a + 6320d = 940$

$16a + 632d = 94$

and

75^{th} term of S is 14.5

① $a + 74d = 14.5$ $(\times 16)$
② $16a + 632d = 94$

$16a + 1184d = 232$ – (solve simultaneously)
$16a + 632d = 94$

$a = -4$

Plug "$a = -4$" back into equation ①

$-4 + 74d = 14.5$
$74d = 18.5$
$d = 0.25$, $a = -4$

given that:

$\frac{X}{2}[2a + (X-1)d] = 171$

Plug in "$d = 0.25$" and "$a = -4$"

$\frac{X}{2}[2(-4) + (X-1)0.25] = 171$

$\frac{X}{2}[-8 + (0.25X - 0.25] = 171$

$\frac{X}{2}[-8.25 + 0.25X] = 171$

$X[-8.25 + 0.25X] = 342$

$-8.25X + 0.25X^2 = 342$

$0.25X^2 - 8.25X - 342 = 0$

$X = -24$ or $X = 57$

→ cannot have a negative number of terms, so $X = 57$

P 7 2 7 9 0 A 0 2 2 2 8

$X =$ 57

(Total for Question 20 is 6 marks)

21 A curve has equation $y = f(x)$

There is only one turning point on the curve.
The coordinates of this turning point are (6, 5)

Write down the coordinates of the turning point on the curve with equation

(a) $y = f(x - 4)$

The change to the function has occured inside the bracket.

Note: think: inside, x, opposite

change is inside the bracket

so the x value is impacted

the change is opposite to what we would expect.

So, (6,5) $\xrightarrow{+4}$ becomes (10,5)

(10 , 5)

(1)

(b) $y = f(3x)$

Inside, x, opposite

(6,5) $\xrightarrow{\div 2}$ (2,5)

(2 , 5)

(1)

(Total for Question 21 is 2 marks)

22 The diagram shows two circles with centre O and a regular pentagon $ABCDE$

Sine rule:
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Sine rule (area):
area $= \frac{1}{2} \times a \times b \times \sin C$

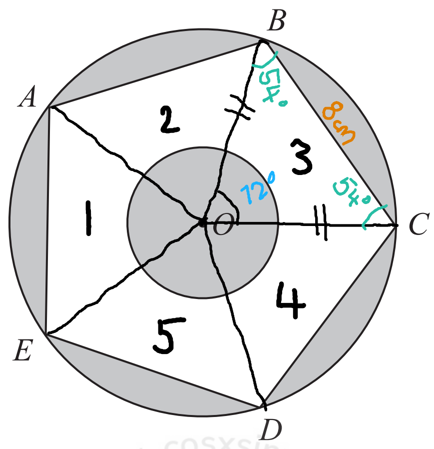


Diagram **NOT** accurately drawn

A, B, C, D and E are points on the larger circle.
The pentagon has sides of length 8 cm.

The diagram is shaded such that

shaded area = unshaded area

Work out the radius of the smaller circle.
Give your answer correct to 3 significant figures.

To find angle BOC: $\frac{360°}{5 \text{ sides of pentagon}} = 72°$

180° in a triangle, the triangle is an isosceles triangle (as BO = CO) so $O\hat{B}C$ and $B\hat{C}O$ will be equal.

$\frac{180 - 72}{2} = 54°$ each

1) Radius of larger circle

using trig: (CAH)
$\cos 54 = \frac{4}{H}$
$\cos 54 \times H = 4$
$H = \frac{4}{\cos 54}$
$H = 6.805206467$

or

using trig (SOH):
$\sin 36 = \frac{4}{H}$
$\sin 36 \times H = 4$
$H = \frac{4}{\sin 36}$
$H = 6.805206467$

or

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$
$a = 8\text{cm}$, $b = OB$
$A = 72°$, $B = 54°$
$\frac{8}{\sin 72} = \frac{OB}{\sin 54}$
$OB = \frac{8 \times \sin 54}{\sin 72}$
$OB = 6.805206467$

2) Area of larger circle:
Area $= \pi r^2$
$\pi \times (6.805...)^2 = 145.489...$

or Area of sector:
$\frac{\theta}{360°} \times \pi r^2$
$= \frac{72}{360} \times \pi (6.805...)^2 = 29.09795584$

3) Area of triangle + therefore pentagon:
$\frac{1}{2} ab \sin C$

$\frac{1}{2} \times 6.805... \times 6.805... \times \sin 72 = 22.02211073$

$22.022... \times 5$ segments which make the pentagon $= 110.1105536$

4) Area large Circle − Area of pentagon + Area small circle = Area of pentagon − Area small circle

$145.489... - 110.11... + \pi r^2 = 110.11... - \pi r^2$

$2\pi r^2 = 110.11... + 110.11... - 145.489...$

$r^2 = \dfrac{110.11... + 110.11... - 145.489...}{2\pi}$

$r^2 = 11.893...$

$r = \sqrt{11.893...}$

$r = 3.448765615$

$r = 3.45$ (3 significant figures)

3.45 .... cm

(Total for Question 22 is 6 marks)

Turn over for Question 23

23 A frustum is made by removing a small square-based pyramid from a similar large squared-based pyramid as shown in the diagram.

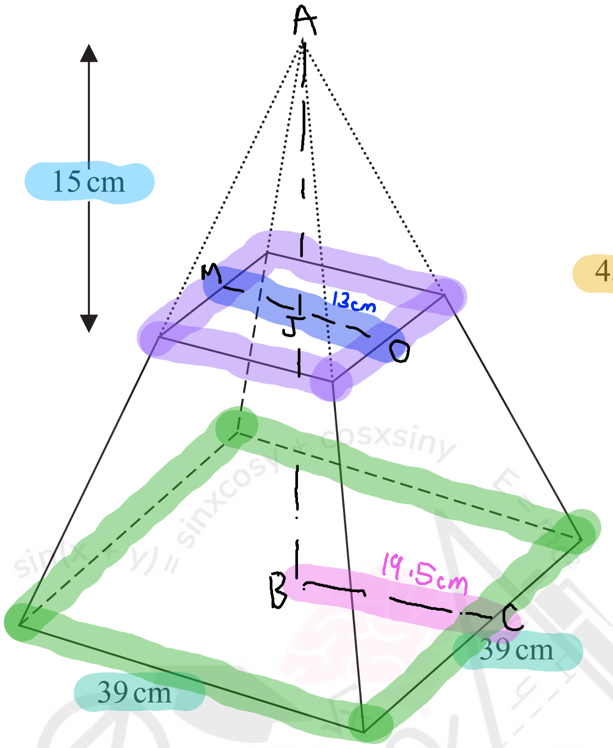


Diagram **NOT** accurately drawn

Scale factor $= \frac{45}{15} = 3$

The height of the small pyramid is 15 cm.
The height of the large pyramid is 45 cm.
The square base of the large pyramid has side length 39 cm.

Work out the **total** surface area of the frustum.
Give your answer correct to the nearest whole number.

Area smaller square + Area bigger square

$(39 \div 3)^2 + 39^2$

$= 169 + 1521$

$= 1690\text{ cm}^2$

Finding the slant heights:

Length $BC = \frac{39}{2} = 19.5\text{cm}$

Using $a^2 + b^2 = c^2$

$45^2 + 19.5^2 = c^2$

$c = \sqrt{45^2 + 19.5^2}$

$c = 49.04334817$

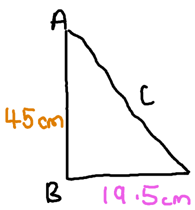
Length $MO = \frac{39}{3} = 13$

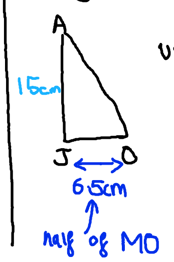
using $a^2 + b^2 = c^2$

$15^2 + 6.5^2 = c^2$

$\sqrt{15^2 + 6.5^2} = c$

$c = 16.34778272$





Surface area of the faces:

$\frac{1}{2} b \times h$

$\frac{1}{2} 39 \times 49.04\ldots = 956.345\ldots \times 4$ faces

$= 3825.381\ldots$

Surface area of the small triangle faces:

$\frac{1}{2} b \times h$

$\frac{1}{2} 13 \times 16.347\ldots$

$= 106.260\ldots \times 4$ faces

$= 425.04\ldots$

Total area:

$1690 + 3825.381\ldots - 425.042\ldots$

$= 5090.339$

$= 5090 \text{ cm}^2$ (to the nearest whole number)

5090 cm^2

(Total for Question 23 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

BLANK PAGE